Higher Twist Scaling Violations

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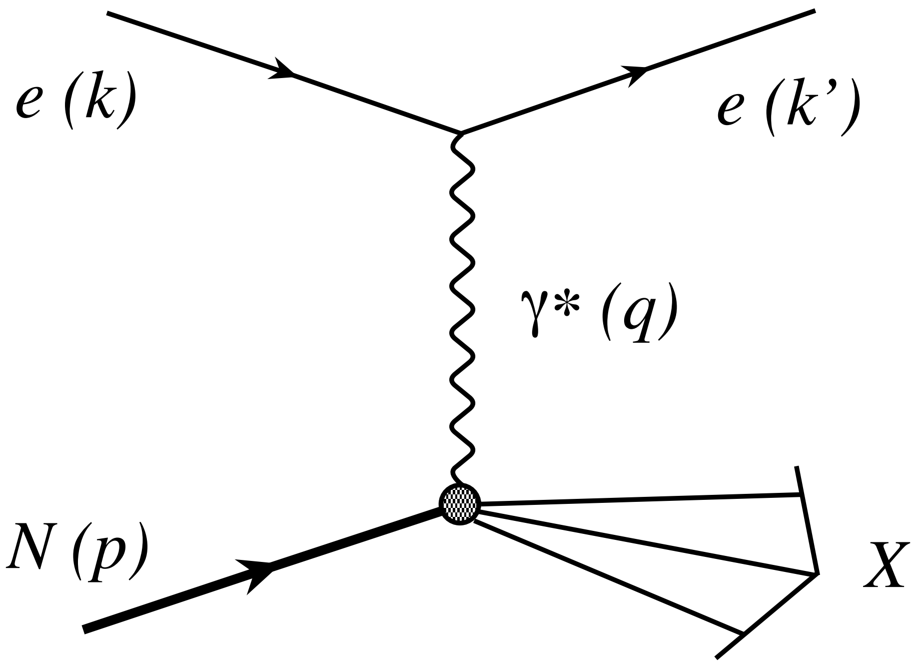
Jefferson Lab - HiX 10/13/10

Outline

- Historical Overview of Scaling Violations in QCD
- Theoretical Foundations
- Perturbative Analysis of Twist-4
- Preliminary Results
- Summary

QCD and Nucleon Structure



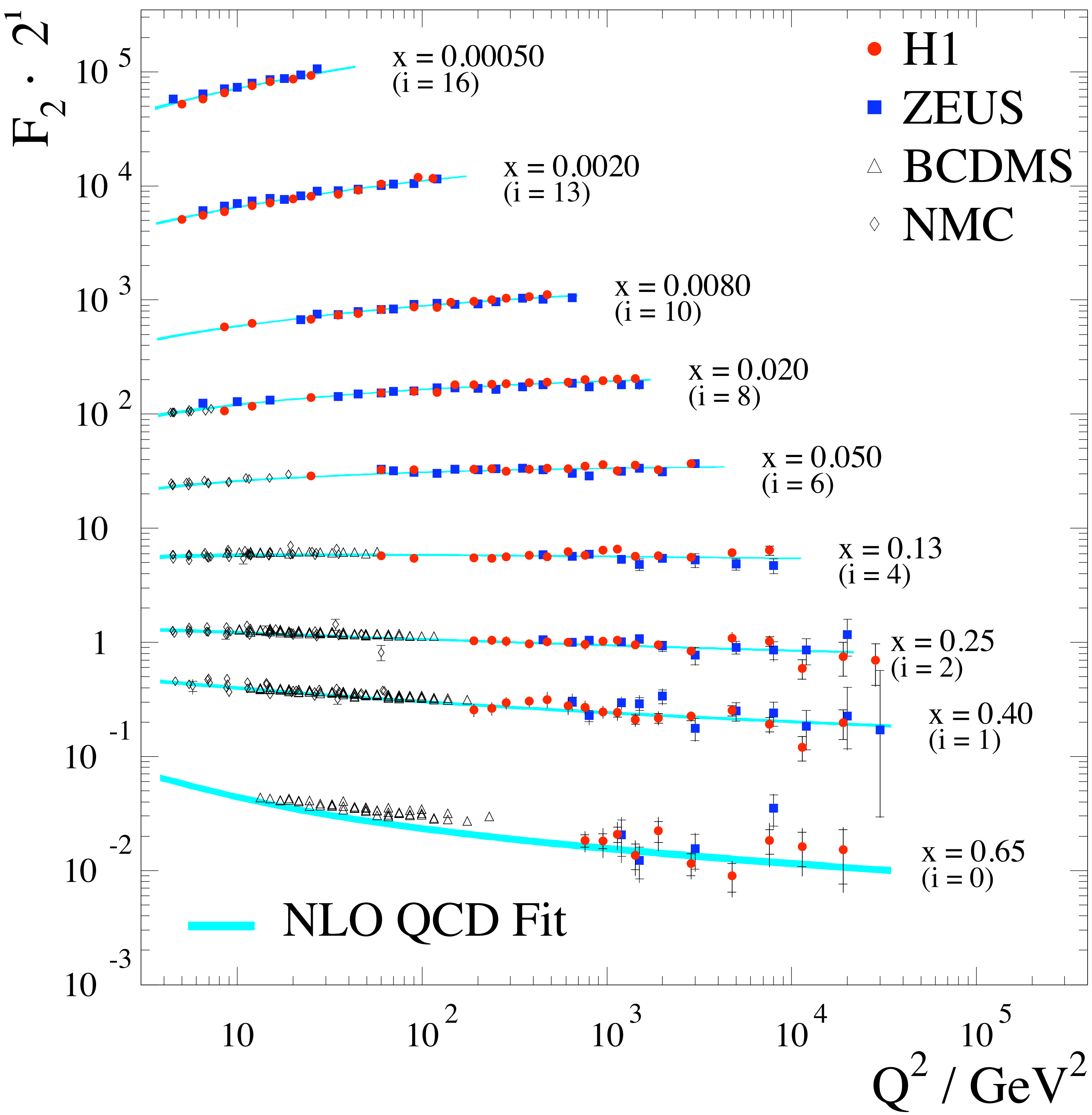
$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left(\frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} + \frac{1}{\nu} F_2(x, Q^2) \right)$$

$$x = \frac{Q^2}{2M\nu}, \qquad \nu = E - E'$$

- SLAC-MIT experiments discovered the proton is a "loose assemblage" of charge.
- Data exhibited Bjorken Scaling.
- Later experiments found deviations to scaling with a logarithmic dependence on Q^2.

Perturbative QCD

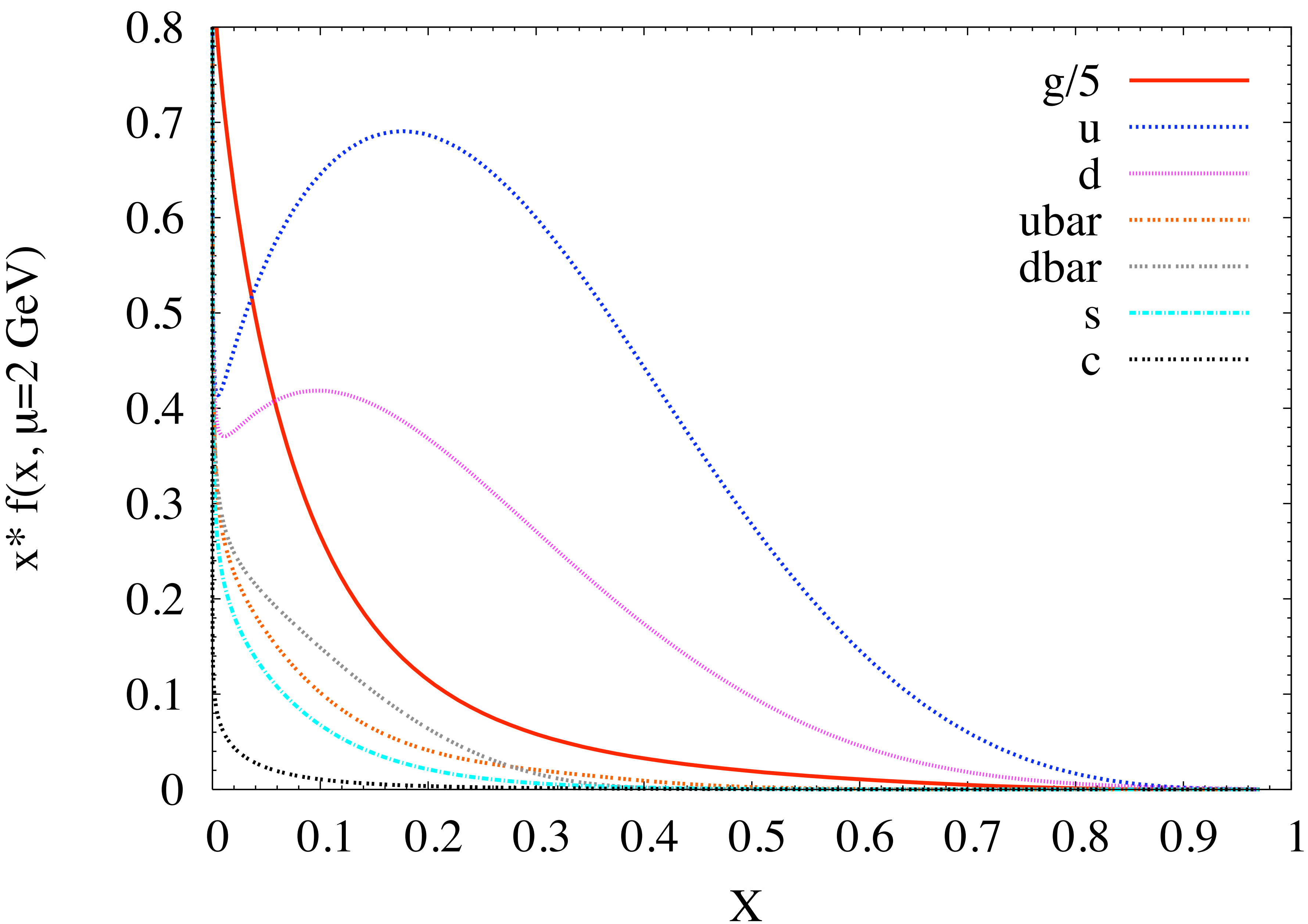
ZEUS Collab., S. Chekanov et al., Phys. Rev. D 70 (2004) 052001

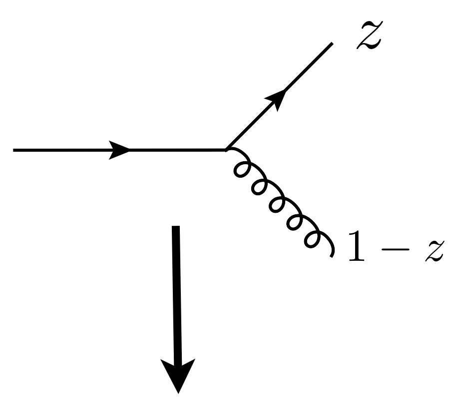


$$F_2(x, Q^2) = x \sum_q e_q^2 (q(x, Q^2) + \bar{q}(x, Q^2))$$

Parton Evolution Liang-Lai et al. - CT10

$$\frac{d}{d\log Q} f_f(x,Q) = \frac{\alpha_s(Q^2)}{\pi} \int_x^1 \frac{dz}{z} \left\{ P_{q\leftarrow q}(z) f_f(\frac{x}{z},Q) + P_{q\leftarrow g}(z) f_g(\frac{x}{z},Q) \right\}$$

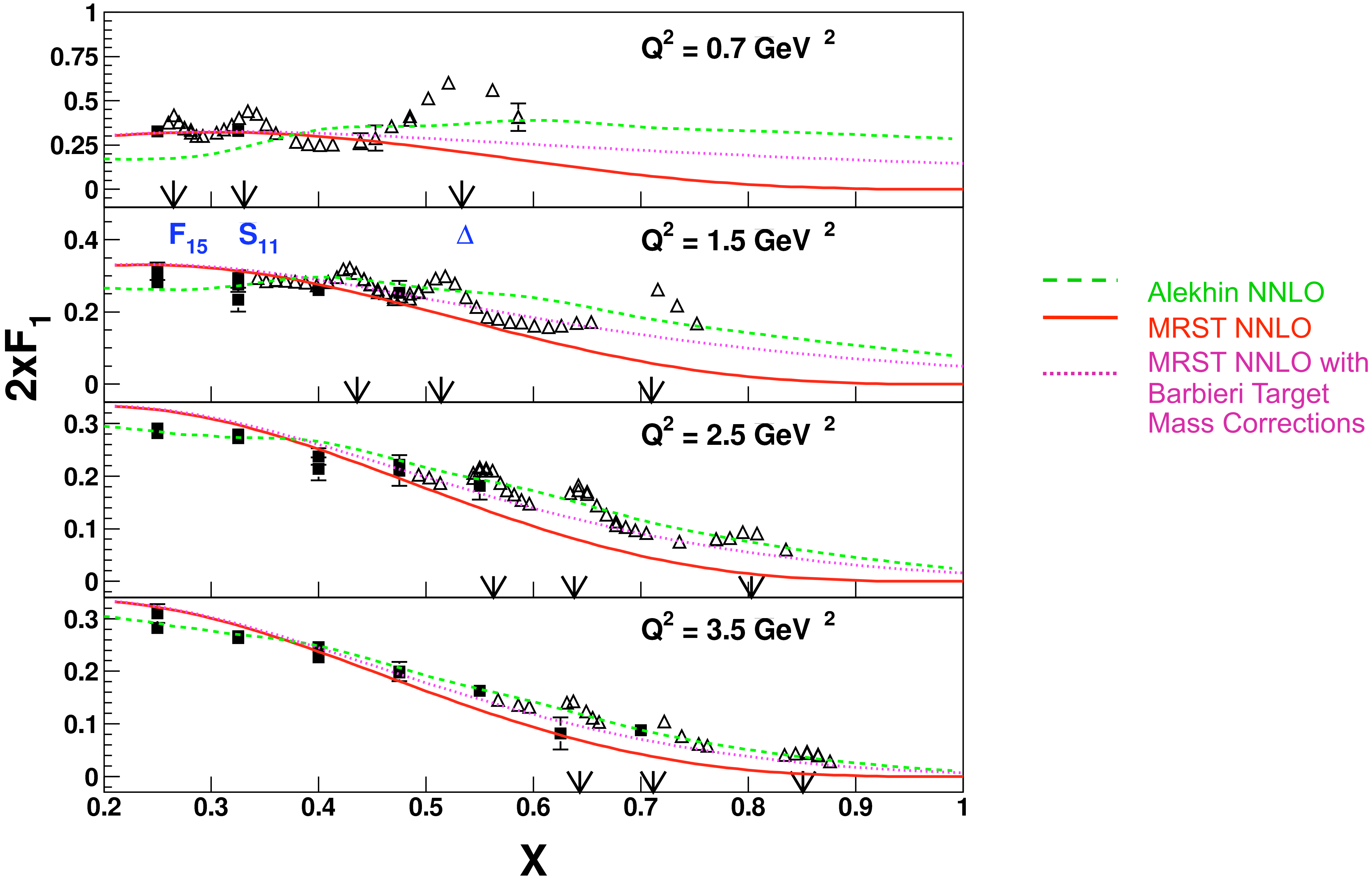




$$P_{qq}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)^2_+} + \frac{3}{2}\delta(1-z)\right]$$

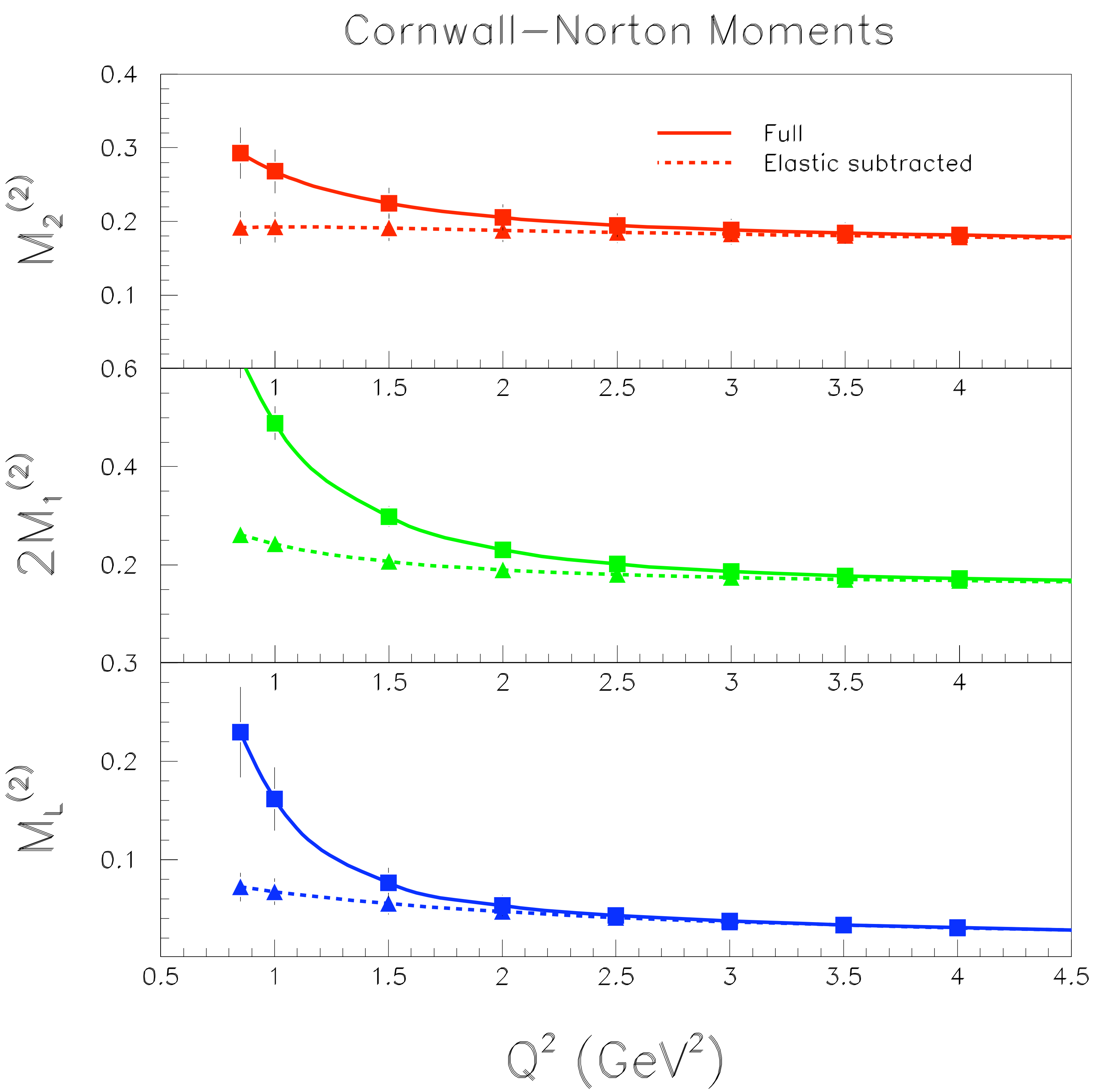
- The parton model gives us an intuitive picture of logarithmic scaling violations!

Beyond the Parton Model Y.Liang et al. JLAB Hall C (E94-110)



Leading Moment Data

(Liang *et al.* JLAB Hall C - CLAS Collaboration) (E94-110)



$$F_2^{\mathrm{EL}} = \frac{(G_E^2 + \tau G_M^2)\delta(x-1)}{1+\tau}$$

$$F_1^{\mathrm{EL}} = G_M^2 \delta(x-1) \qquad \tau = \frac{q^2}{4M_p^2}$$

$$F_L^{\mathrm{EL}} = G_E^2 \delta(x-1)$$

Elastic Contributions

Cornwall-Norton Moments

- Bjorken-x weighted integral

$$M_n(Q^2) = \int_0^1 dx_B \;\; x_B^{n-2} F_2(x_B, Q^2)$$

- Which can be analyzed in terms of the operator product expansion

Twist

$$M_n(Q^2, g, \mu) \approx \sum_k \left(\frac{1}{Q^2}\right)^{\frac{\tau-2}{2}} \tilde{c}_k^n(g,\mu) \;\; A_k^{(n)}$$

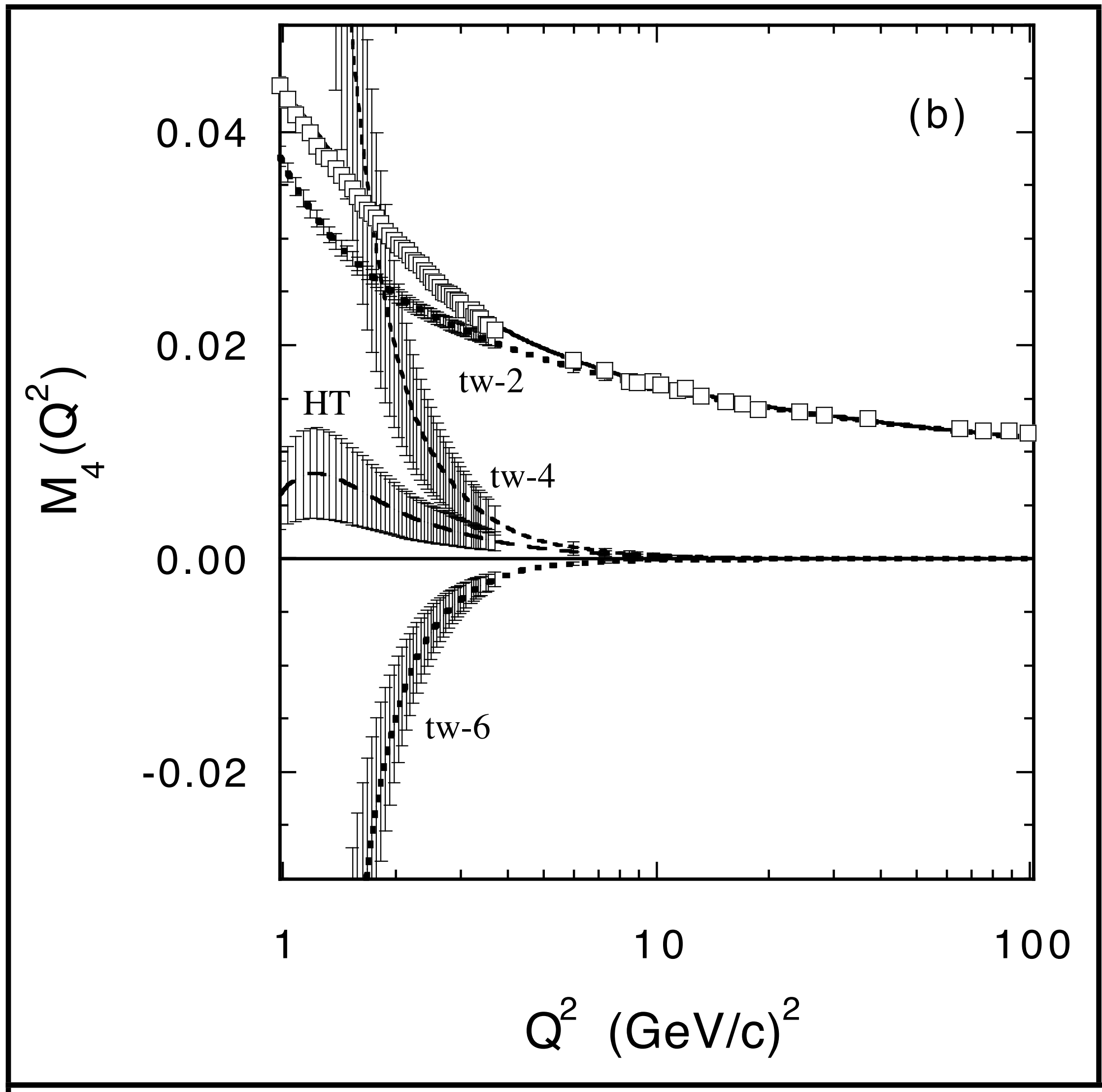
Non-Perturbative Matrix Element

Wilson Coefficient

- De Rujula, Georgi & Politzer originally explained Duality by placing bounds on the higher-twist matrix elements Ann. of Phy 353 (315-353) 1977

Cancellation of Higher Twist?

(Liang *et al.* JLAB Hall C - CLAS Collaboration)



$$M_n(Q^2) = \eta_n(Q^2) + a_n^{(4)} \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right]^{\gamma_n^{(4)}} \frac{\mu^2}{Q^2} + a_n^{(6)} \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right]^{\gamma_n^{(6)}} \frac{\mu^4}{Q^4}$$

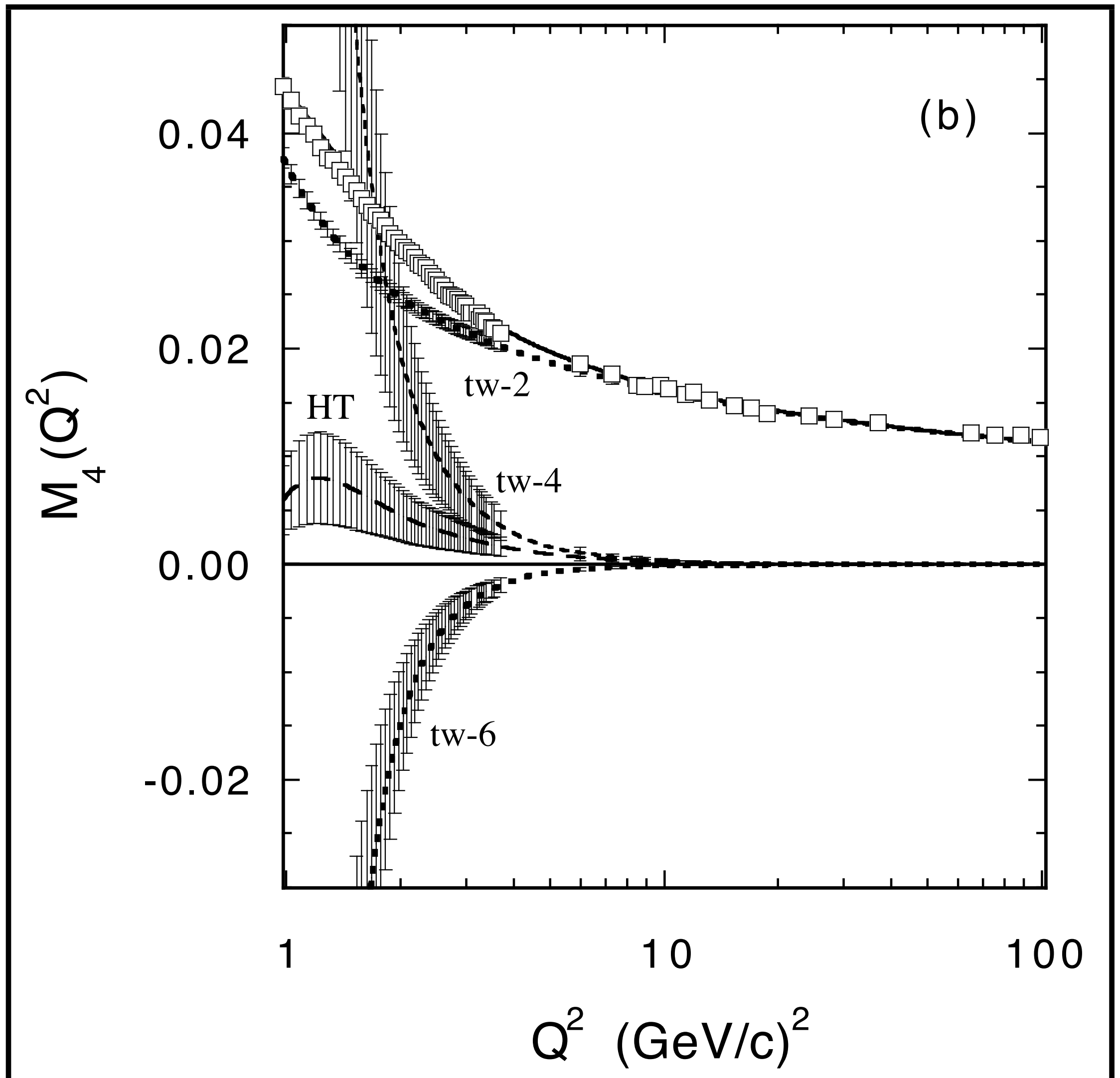
Leading Twist

Twist 4

Effective gamma

Cancellation of Higher Twist?

(Liang *et al.* JLAB Hall C - CLAS Collaboration)



$$M_n(Q^2) = \eta_n(Q^2) + a_n^{(4)} \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right]^{\gamma_n^{(4)}} \frac{\mu^2}{Q^2} + a_n^{(6)} \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right]^{\gamma_n^{(6)}} \frac{\mu^4}{Q^4}$$

Leading Twist

Twist 4

Effective gamma

Many Operators Contribute!

$$\Delta \cdot Q_n^{1(k,\ell)} = g\bar\psi_R \overleftarrow{d}^{\ell} \overrightarrow{d}^{k} \psi_R \bar\psi_R \overrightarrow{d}^{n-2-k-\ell} \psi_R,$$

$$\Delta \cdot Q_n^{2(k,\ell)} = g\bar\psi_R \tau_a \overleftarrow{d}^{\ell} \overrightarrow{d}^{k} \psi_R \bar\psi_R \overrightarrow{d}^{n-2-k-\ell} \tau_a \psi_R,$$

$$\Delta \cdot Q_n^{3(k,\ell)} = g\bar\psi_R \overleftarrow{d}^{\ell} \overrightarrow{d}^{k} \psi_R \bar\psi_L \overrightarrow{d}^{n-2-k-\ell} \psi_L,$$

$$\Delta \cdot Q_n^{4(k,\ell)} = g\bar\psi_R \tau_a \overleftarrow{d}^{\ell} \overrightarrow{d}^{k} \psi_R \bar\psi_L \overrightarrow{d}^{n-2-k-\ell} \tau_a \psi_L,$$

$$\Delta \cdot Q_n^{5(k,\ell)} = g\bar\psi_L \overleftarrow{d}^{\ell} \overrightarrow{d}^{k} \psi_L \bar\psi_L \overrightarrow{d}^{n-2-k-\ell} \psi_L,$$

$$\Delta \cdot Q_n^{6(k,\ell)} = g\bar\psi_L \tau_a \overleftarrow{d}^{\ell} \overrightarrow{d}^{k} \psi_L \bar\psi_L \overrightarrow{d}^{n-2-k-\ell} \tau_a \psi_L,$$

$$\Delta \cdot Q_n^{7(k,\ell)} = \bar\psi \overleftarrow{d}^{k} \gamma_5 \overrightarrow{d}^{n-1-k} \psi,$$

$$\Delta \cdot Q_n^{8(k,\ell)} = i\bar\psi \overleftarrow{d}^{k} \overrightarrow{d}^{n-1-k} \psi,$$

$$\Delta \cdot O_n^{G1a(k,\ell)} = \mathrm{Tr}[f_\alpha \overrightarrow{d}^{n-4-k-\ell} f^\alpha]\mathrm{Tr}[\overrightarrow{d}^{k} f_\beta \overrightarrow{d}^{\ell} f^\beta]$$

$$\Delta \cdot O_n^{G3} = \mathrm{Tr}[G^{\alpha\beta} \overrightarrow{d}^{n} G_{\alpha\beta}]$$

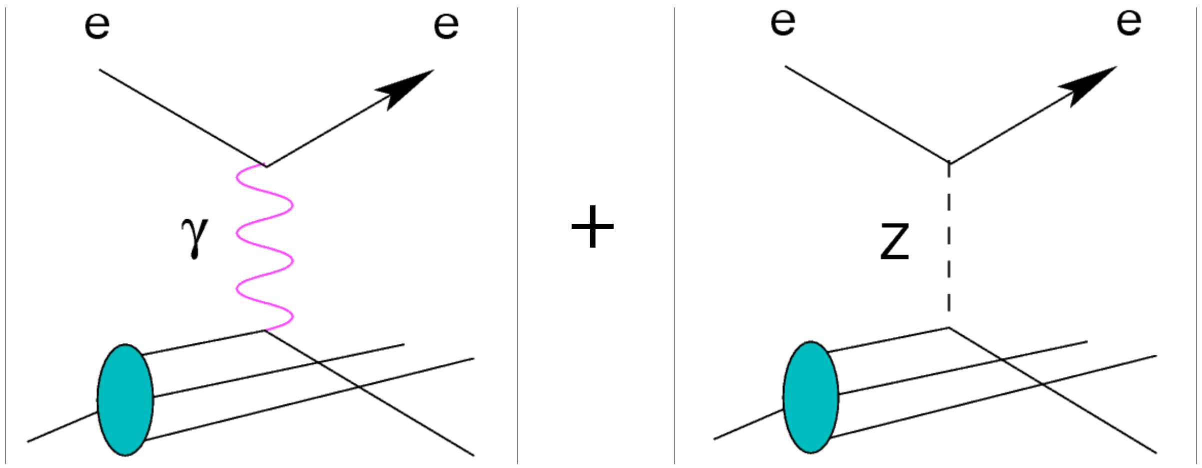
Summary of Goals

- Complete a detailed study of the RG evolution of twist-4 operators, reducing the number of fit parameters for higher twist effects in DIS.

- Through the combination of data and lattice simulations we hope to provide a good first step toward a systematic program of analyzing higher twist correlations in the nucleon.

- More generally, a better understanding of HT can inform electroweak observables. Nuclear effects must be well understood before claims of new physics can be made.

PVDIS - Electron/Deuteron Asymmetry



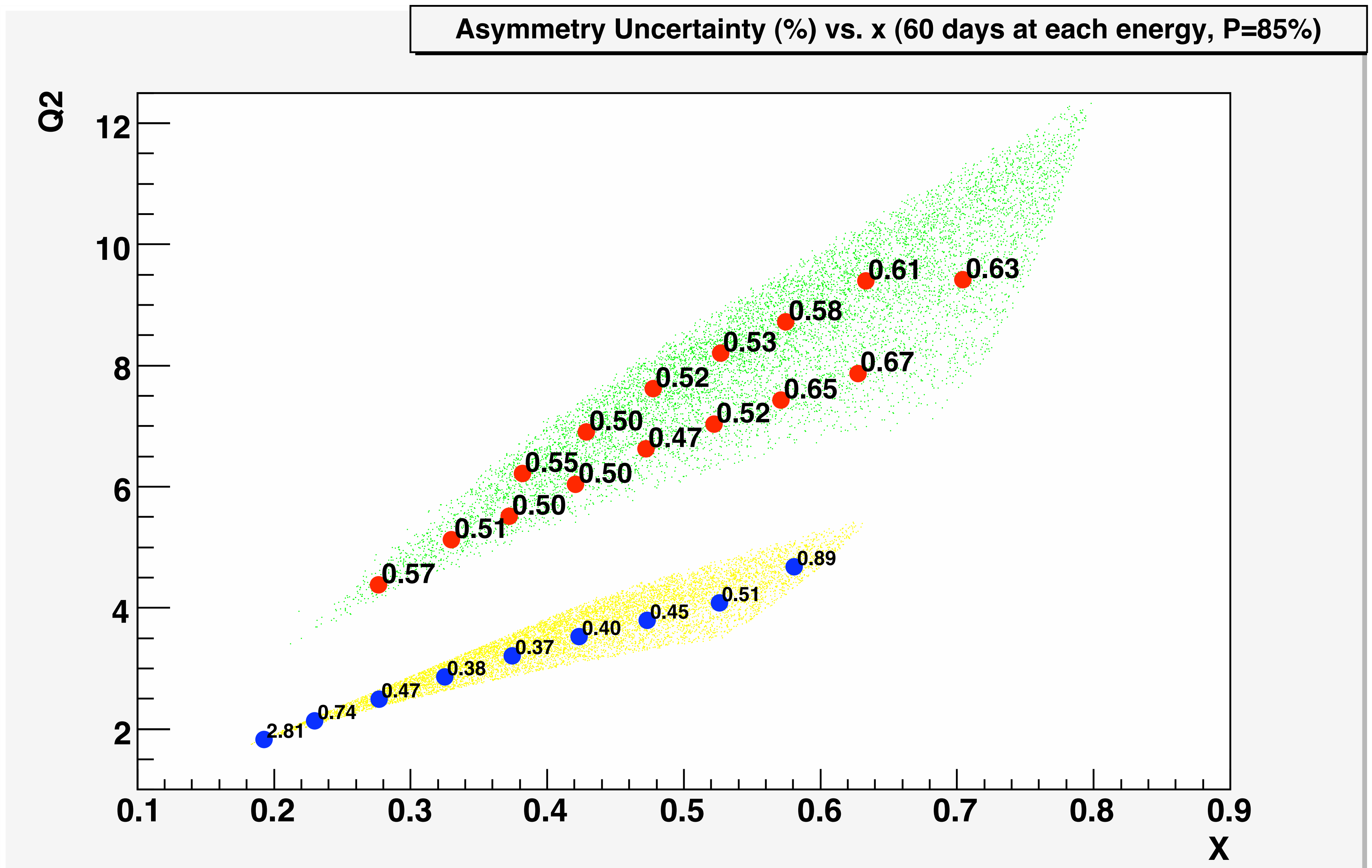
$$\mathcal{A}_{\mathrm{RL}} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

- The focus has shifted from the SM WNC theory to detecting hints of physics beyond the SM

- 12 GeV program to begin at JLab in 2014
  - Qweak
    (W.T.H. Van Oers)
  - SOLID, 6 GeV, and 12, GeV experiments
    (P. Souder, P. Reimer)

SOLID

- SOLID plans to measure the asymmetry at a percent level over a wide kinematic range.



Projected data with errors for SOLID
(P. Souder)

PVDIS - Electron/Deuteron Asymmetry

$$\mathcal{A}_{\rm RL} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

- Precision PVDIS must control hadronic uncertainties: TMC, CSV, sea quark distributions, higher twist effects.

- HT effects in the first term of the asymmetry are given in terms of a single four quark matrix element!
  (Cahn-Gilman; Bjorken, Wolfenstein; Hobbs, Melnitchouk; Mantry, Musolf, Sacco)

$$\boxed{\mathcal{O}_{ud}^{\mu\nu}(x) = \frac{1}{2}[\bar{u}(x)\gamma^{\mu}u(x)d(0)\gamma^{\nu}d(0) + (u \leftrightarrow d)]}$$

- The RG evolution of four-quark operators facilitates an extraction of higher twist matrix elements.

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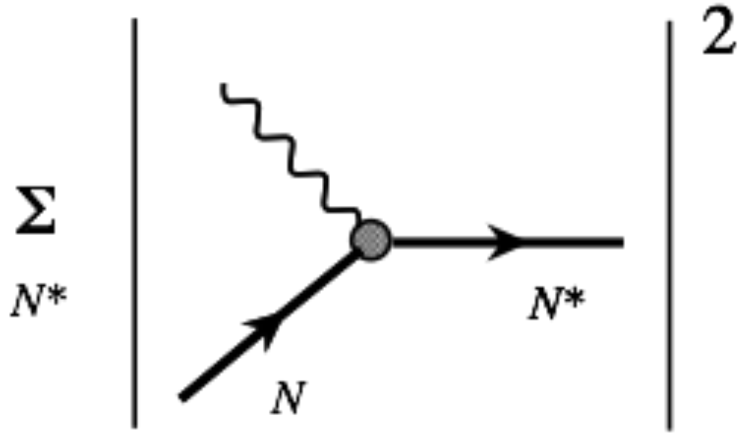
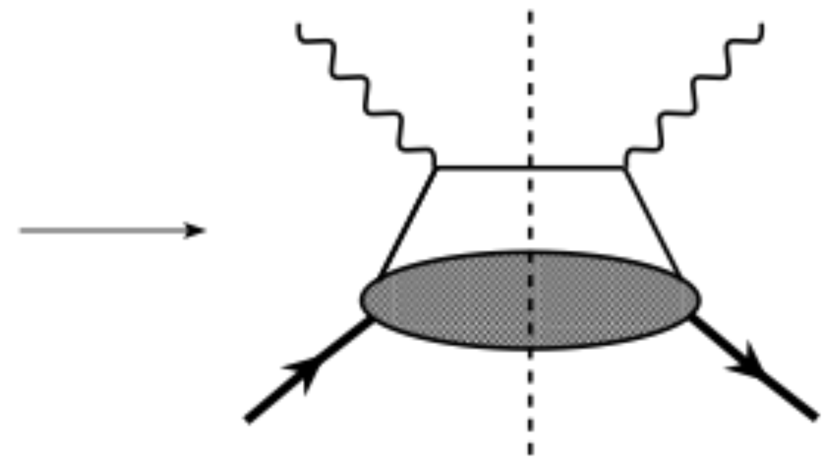
Operator Product Expansion Wilson, Phys Rev 179 (1979)

Optical Theorem



$\sim L_{\mu\nu} W^{\mu\nu}$

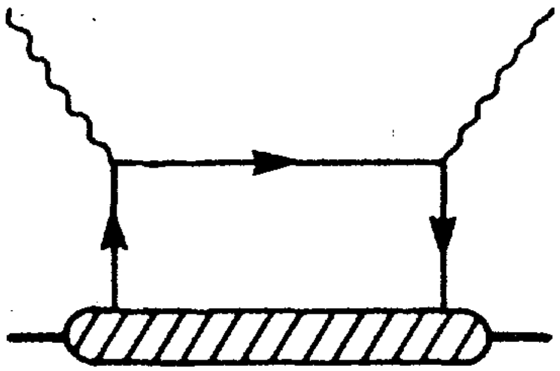


$$\mathrm{T}\{J_\mu(x) J_\nu(0)\} \sim \Gamma_{\mu\nu} \sum_{n,k} C_k^{(n)}(x^2) \mathcal{O}_k^{(n)}(0)$$

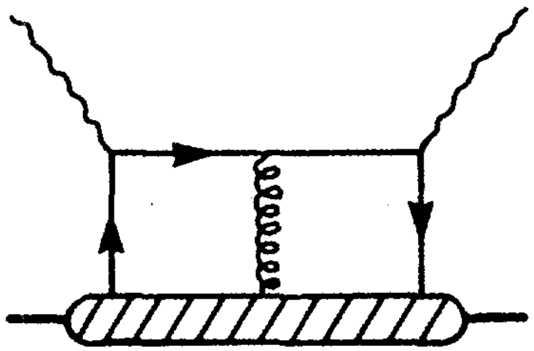
Wilson-Coefficient

Local Operator

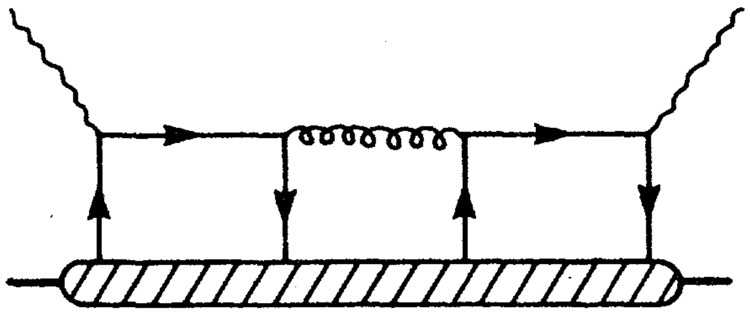
Higher Twist



Twist-2



Quark-gluon correlation



Quark-quark correlation

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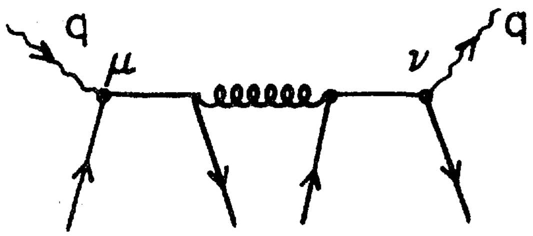
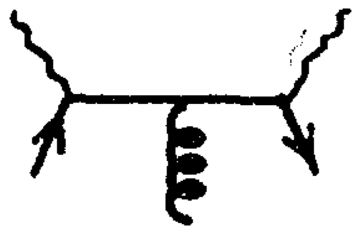
The Operator Basis R.L. Jaffe & M. Soldate - Phys. Rev. D V26 No.1

- 12 Operators appear at twist-4, which can be divided into three groups:



4-Quark $\triangle \cdot Q_n^{1(k,l)} = g[\bar\psi_R \not\Delta \overleftarrow{d}^{\,l} \overrightarrow{d}^{\,k} \psi_R][\bar\psi_R \not\Delta \overrightarrow{d}^{\,n-2-k-l}\psi_R]$

2-Quark $\triangle \cdot Q_n^{8(k)} = i\bar\psi \overleftarrow{d}^{\,k} \not f \overrightarrow{d}^{\,n-1-k}\psi$



Gluonic $\triangle \cdot G_n^{(k,l)} = \mathrm{Tr}[f_\alpha \overrightarrow{d}^{\,n-k-l} f^\alpha \overrightarrow{d}^{\,k} f_\beta \overrightarrow{d}^{\,l} f^\beta]$

The Anomalous Dimension Matrix

- The anomalous dimension takes the following form

$$\gamma \sim \begin{pmatrix} \text{Quark} & \text{Quark} \to \text{Glue} \\ \text{Glue} \to \text{Quark} & \text{Glue} \end{pmatrix}$$

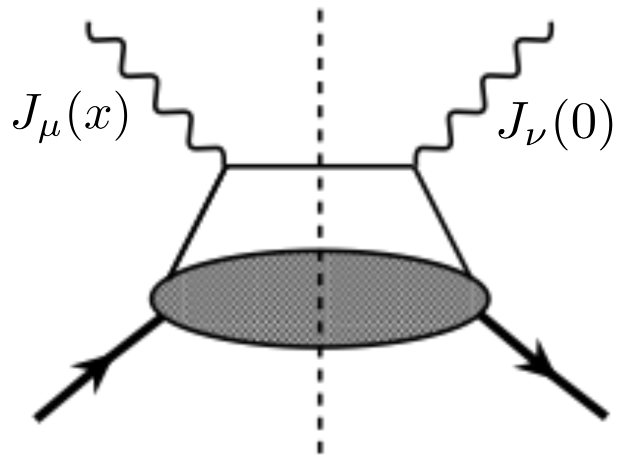
- The quark sector can be further decomposed

$$\gamma_{\text{Quark}} \sim \begin{pmatrix} 4Q \to 4Q & \mathbf{0} \\ 4Q \to 2Q & 2Q \to 2Q \end{pmatrix}_{\text{f,I}}$$

Flavor/Isospin

SU(3) - Flavor Decomposition

- Each current sits in the octet representation of SU(3)-flavor.



$$J^\mu(x) = \bar{\psi}(x)\gamma^\mu \frac{1}{2}\left(\lambda_f^3 + \frac{1}{\sqrt{3}}\lambda_f^8\right)\psi(x) \quad \text{for} \quad SU(3)_f$$

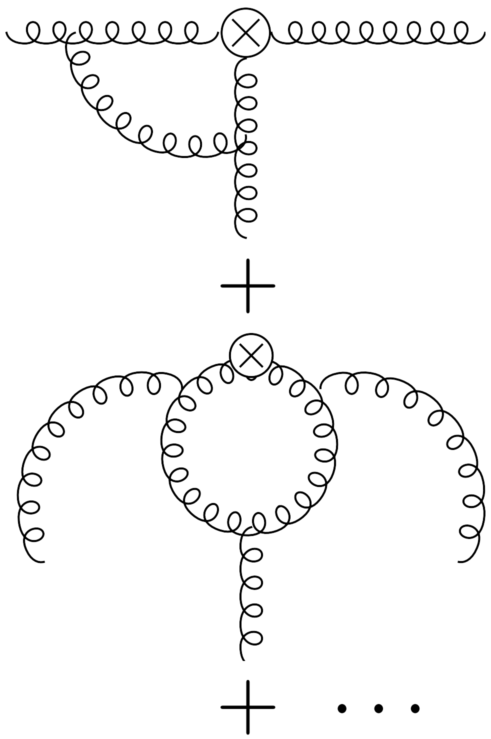
- The direct product of these octets contains multiple representations. (I3 = Y = 0)

$$8 \otimes 8 = 27 \oplus 10 \oplus \bar{10} \oplus 8_1 \oplus 8_2 \oplus 1$$

Flavor Singlet Sector

$$\gamma \sim \begin{pmatrix} \text{Quark} & \text{Quark} \to \text{Glue} \\ \text{Glue} \to \text{Quark} & \text{Glue} \end{pmatrix}$$

- A work in progress....



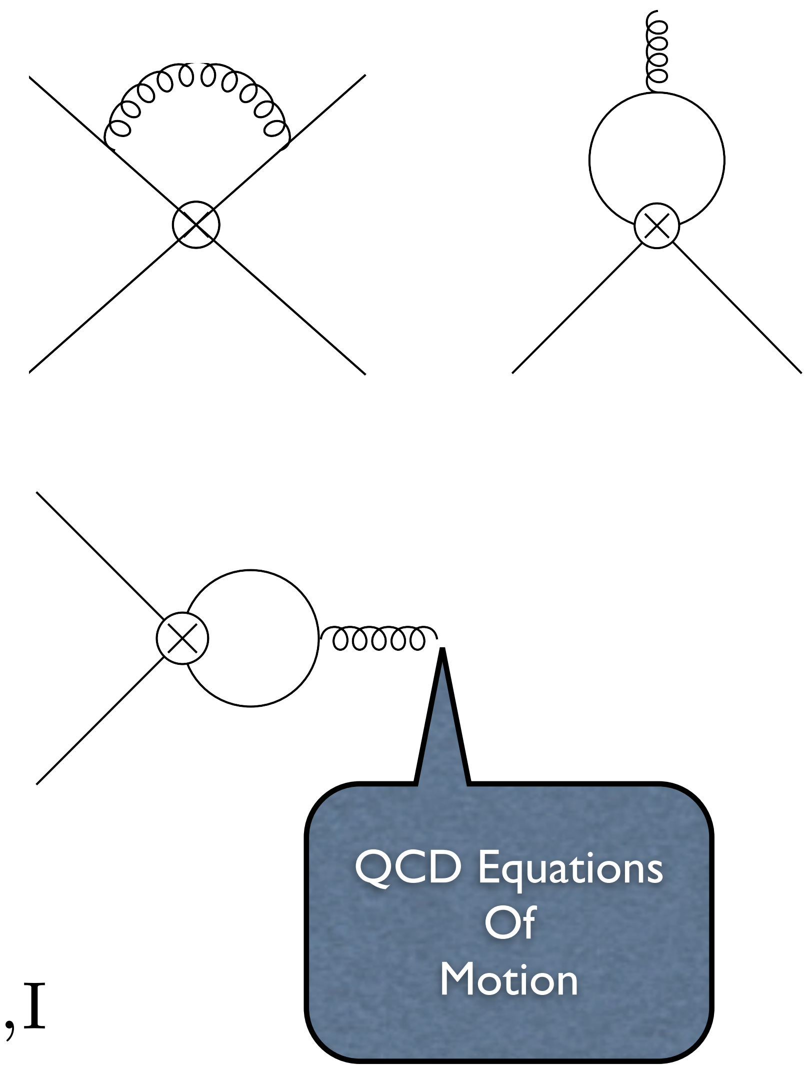
$$\gamma_{\text{Glue}} \sim \begin{pmatrix} G_1 \to G_1 & G_1 \to G_2 \\ G_2 \to G_1 & G_2 \to G_2 \end{pmatrix}_{\text{f,I}}$$

Four Quark Sector: 27-Plet (Gottlieb, Okawa)

$$\tilde{Q}_{27,\mathrm{I}} \sim \sum_f C_f \left(\bar{\psi}_{\mathrm{L,R}}\Gamma_\mu \psi_{\mathrm{L,R}}\right)_f \left(\bar{\psi}_{\mathrm{L,R}}\Gamma_\nu \psi_{\mathrm{L,R}}\right)_f$$

- Large, but sparse matrix:

$$\gamma_{\mathrm{I}}^{27} \sim \begin{pmatrix} & & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ 0 & 0 & & & 0 & 0 \\ 0 & 0 & & & 0 & 0 \\ 0 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & & \end{pmatrix} \begin{pmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \\ \tilde{Q}_3 \\ \tilde{Q}_4 \\ \tilde{Q}_5 \\ \tilde{Q}_6 \end{pmatrix}_{27,\mathrm{I}}$$



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Running of 27, I=1 Preliminary

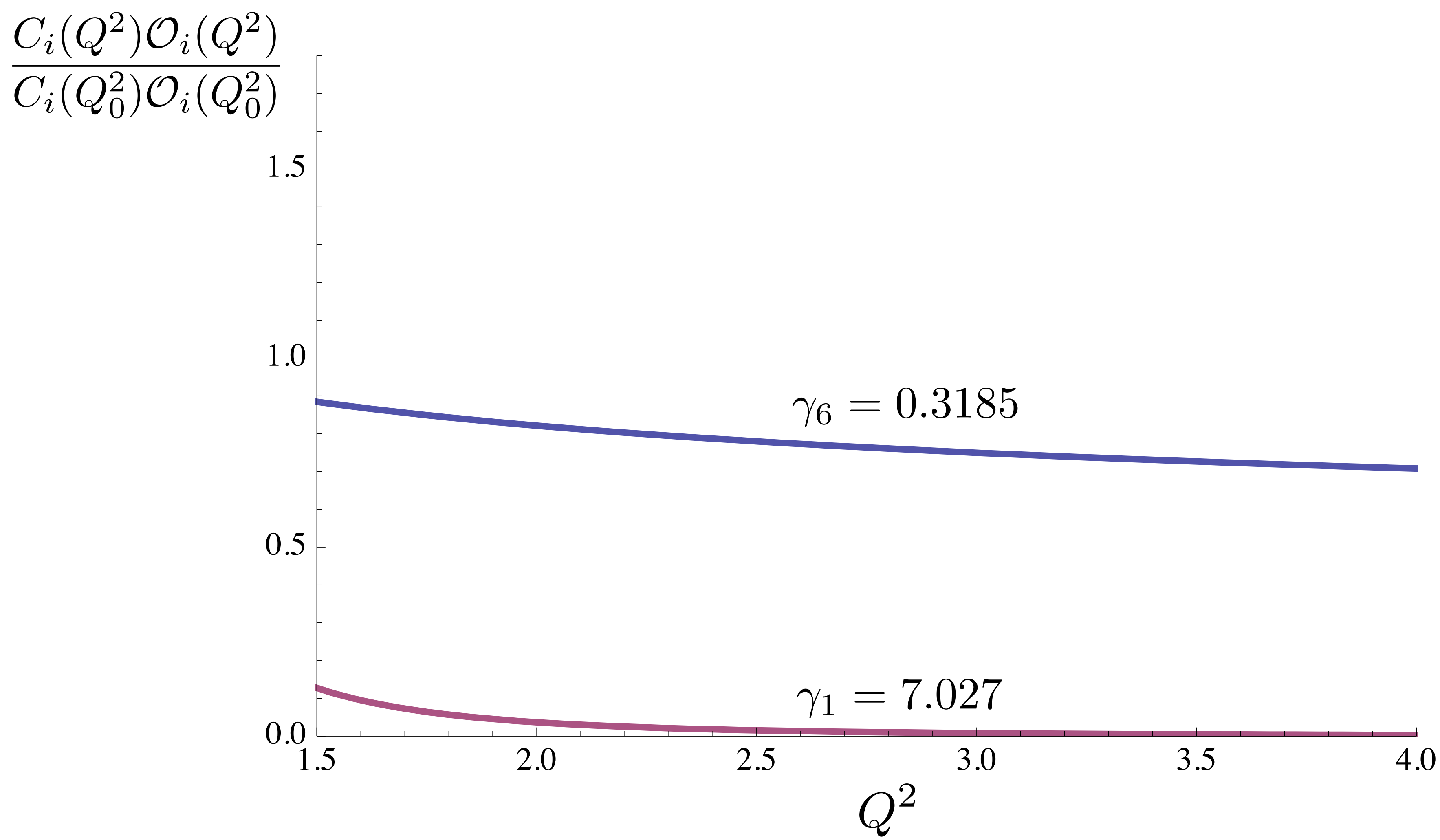
- As a simplistic example, consider just the exponential factor

$$M_n\left(Q^2\right) \approx \sum_i \left(\frac{1}{Q^2}\right)^{\frac{\tau-2}{2}} \tilde{c}_j^n\left(Q^2, g(t), \mu\right) \exp\left[-\int_0^t \gamma_{ij}^{(n)}(\bar{g}(t'))dt'\right] A_i^n$$

- Diagonalize gamma, giving a linear combination of 6 operators and 6 eigenvalues

$$\frac{C_i(Q^2)\mathcal{O}_i(Q^2)}{C_i(Q_0^2)\mathcal{O}_i(Q_0^2)} \sim \exp\left[-\int_0^t \gamma_j(\bar{g}(t'))dt'\right]$$

Running of Wilson-Coefficients Preliminary



Leading Moment (n=2)

$$M_n\left(Q^2\right) \approx \sum_i \left(\frac{1}{Q^2}\right)^{\frac{\tau-2}{2}} \tilde{c}_j^n\left(Q^2, g(t), \mu\right) \exp\left[-\int_0^t \gamma_{ij}^{(n)}(\bar{g}(t'))dt'\right] A_i^n$$

$$\frac{C_i(Q^2)\mathcal{O}_i(Q^2)}{C_i(Q_0^2)\mathcal{O}_i(Q_0^2)} \sim \exp\left[-\int_0^t \gamma_j(\bar{g}(t'))dt'\right]$$

Outlook

- Complete the singlet sector of the twist-4 anomalous dimension.
- Include tree level Wilson Coefficients for the full anomalous dimension.
- Provide a detailed analysis of the RG evolution of twist four.
- In the future, we hope to extend this analysis to higher moments.
- Can be done using non-local operator renormalization technique - Balitsky, Braun et al. Nuc Phys B 807, 2009.

Summary

- We hope that these calculations combined with lattice estimates of twist four matrix elements will provide a complete program for systematic study of higher twist.
- RG analysis of twist four can aid in extractions of twist-4 matrix elements.
- The higher moments, sensitive to higher x, are accessible using non-local operator renormalization.
- Please stay tuned for future results!

Thank you!

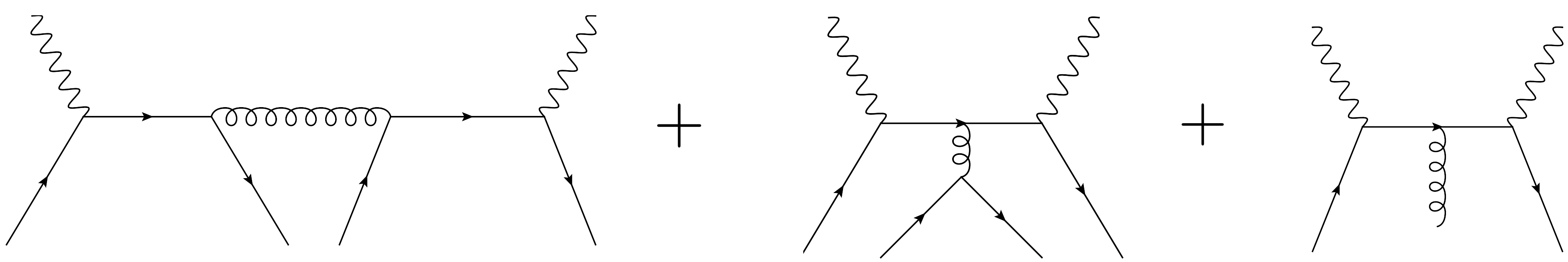
Backup Slides

Quark Mixing: Flavor Octet

- The SU(3)-flavor reduction provides a nice way to organize the quark mixings.

$$\gamma_{\text{Quark}} \sim \begin{pmatrix} 4Q \to 4Q & \mathbf{0} \\ 2Q \to 4Q & 2Q \to 2Q \end{pmatrix}_{\text{f,I}}$$

- Mixings among two/four quark operators are more involved.



$(\bar{\psi}Q\psi)(\bar{\psi}Q\psi)$ + $(\bar{\psi}Q^2\psi)(\bar{\psi}\psi)$ + $(\bar{\psi}Q^2\psi)$

$$\bar{\psi}Q\psi\,\bar{\psi}Q\psi = \sqrt{\frac{2}{3}}O^{27}_{I=2} + \frac{2}{\sqrt{10}}O^{27}_{I=1} + \frac{2}{\sqrt{30}}O^{27}_{I=0} + \frac{2}{\sqrt{15}}O^{8_1}_{I=1} + \frac{2}{3\sqrt{5}O^{8_1}_{I=0}} - \frac{\sqrt{2}}{3}O^{1}_{I=0}$$

Example Analysis